# PETSc <br> Portable, Extensible Toolkit for Scientific Computation 

Karl Rupp<br>rupp@mcs.anl.gov

Mathematics and Computer Science Division
Argonne National Laboratory

Tutorial at the HPC Symposium 2013

April 10th, 2013
U.S. DEPARTMENT OF

ENERGY

## Before we start...

## Ask Questions

Tell me if you do not understand
Ask for further details

## Table of Contents

## About PETSc

First Steps

Application Integration

Profiling

PETSc and GPUs

## PETSc

## About PETSc

## PETSc Origins

## PETSc was developed as a Platform for Experimentation

We want to experiment with different
Models
Discretizations
Solvers
Algorithms

These boundaries are often blurred...

## Domein Deaomposition <br> Paraille Mimfileved Melinuis fop <br> Elipilie Paplial Dificeraniial Equalions <br> 

## Timeline

> MPI-1 MPI-2


## PETSc

## Portable Extensible Toolkit for Scientific Computing

## Architecture

tightly coupled (e.g. XT5, BG/P, Earth Simulator)
loosely coupled such as network of workstations
GPU clusters (many vector and sparse matrix kernels)
Software Environment
Operating systems (Linux, Mac, Windows, BSD, proprietary Unix)
Any compiler
Usable from C, C++, Fortran 77/90, Python, and MATLAB
Real/complex, single/double/quad precision, 32/64-bit int
System Size
500B unknowns, $75 \%$ weak scalability on Jaguar (225k cores) and Jugene (295k cores)
Same code runs performantly on a laptop
Free to everyone (BSD-style license), open development

## PETSc

## Portable Extensible Toolkit for Scientific Computing

## Philosophy: Everything has a plugin architecture

Vectors, Matrices, Coloring/ordering/partitioning algorithms
Preconditioners, Krylov accelerators
Nonlinear solvers, Time integrators
Spatial discretizations/topology*

## Example

Vendor supplies matrix format and associated preconditioner, distributes compiled shared library.
Application user loads plugin at runtime, no source code in sight.

## PETSc

## Portable Extensible Toolkit for Scientific Computing

Toolset
algorithms
(parallel) debugging aids
low-overhead profiling

## Composability

try new algorithms by choosing from product space
composing existing algorithms (multilevel, domain decomposition, splitting)

## Experimentation

Impossible to pick the solver a priori
PETSc's response: expose an algebra of composition
keep solvers decoupled from physics and discretization

## PETSc

## Portable Extensible Toolkit for Scientific Computing

## Computational Scientists

PyLith (CIG), Underworld (Monash), Magma Dynamics (LDEO, Columbia), PFLOTRAN (DOE), SHARP/UNIC (DOE)
Algorithm Developers (iterative methods and preconditioning)
Package Developers
SLEPc, TAO, Deal.II, Libmesh, FEniCS, PETSc-FEM, MagPar, OOFEM, FreeCFD, OpenFVM

Funding
Department of Energy
SciDAC, ASCR ISICLES, MICS Program, INL Reactor Program
National Science Foundation
CIG, CISE, Multidisciplinary Challenge Program
Documentation and Support
Hundreds of tutorial-style examples
Hyperlinked manual, examples, and manual pages for all routines
Support from petsc-maint@mcs.anl.gov

## The Role of PETSc

Developing parallel, nontrivial PDE solvers that deliver high performance is still difficult and requires months (or even years) of concentrated effort.
PETSc is a toolkit that can ease these difficulties and reduce the development time, but it is not a black-box PDE solver, nor a silver bullet.

- Barry Smith


## The Role of PETSc

You want to think about how you decompose your data structures, how you think about them globally. [...]
If you were building a house, you'd start with a set of blueprints that give you a picture of what the whole house looks like. You wouldn't start with a bunch of tiles and say. "Well 'll put this tile down on the ground, and then l'll find a tile to go next to it."
But all too many people try to build their parallel programs by creating the smallest possible tiles and then trying to have the structure of their code emerge from the chaos of all these little pieces. You have to have an organizing principle if you're going to survive making your code parallel.
— Bill Gropp
— http://www.rce-cast.com/Podcast/rce-28-mpich2.html

## PETSc

First Steps

## PETSc

## Obtaining PETSc

http://mcs.anl.gov/petsc, download tarball
Linux Package Managers
Git: https://bitbucket.org/petsc/petsc
Mercurial: https://bitbucket.org/petsc/petsc-hg

## Installing PETSc

```
$> export PETSC_DIR=$PWD PETSC_ARCH=mpich-gcc-dbg
```

```
$> ./configure --with-shared-libraries
    --with-blas-lapack-dir=/usr
    --download-{mpich,ml,hypre}
```

```
$> make all test
```


## PETSc External Packages

Most packages can be automatically
Downloaded
Configured and Built (in \$PETSC_DIR/externalpackages) Installed with PETSc

Currently works for
petsc4py
PETSc documentation utilities (Sowing, Igrind, c2html)
BLAS, LAPACK, BLACS, ScaLAPACK, PLAPACK
MPICH, MPE, OpenMPI
ParMetis, Chaco, Jostle, Party, Scotch, Zoltan
MUMPS, Spooles, SuperLU, SuperLU_Dist, UMFPack, pARMS
PaStiX, BLOPEX, FFTW, SPRNG
Prometheus, HYPRE, ML, SPAI
Sundials
Triangle, TetGen, FIAT, FFC, Generator
HDF5, Boost

## PETSc Pyramid

## PETSc Structure

## PETSc PDE Application Codes

ODE Integrators
Nonlinear Solvers
Linear Solvers
Preconditioners + Krylov Methods Object-Oriented
Matrices, Vectors, Indices


## Profiling Interface

Computation and Communication Kernels MPI, MPI-IO, BLAS, LAPACK

## Main Routine



## PETSc Objects

## Sample Code

```
Mat A;
PetscInt m,n,M,N;
MatCreate(comm, &A);
MatSetSizes(A,m,n,M,N) ; /* or PETSC_DECIDE */
MatSetOptionsPrefix(A,"foo_");
MatSetFromOptions(A);
/* Use A */
MatView(A,PETSC_VIEWER_DRAW_WORLD);
MatDestroy(A);
```


## Remarks

Mat is an opaque object (pointer to incomplete type)
Assignment, comparison, etc, are cheap
What's up with this "Options" stuff?
We will discuss this later...

## Basic PetscObject Usage

Every object in PETSc supports a basic interface

| Function | Operation |
| ---: | :--- |
| Create() | create the object |
| Get/SetName() | name the object |
| Get/SetType() | set the implementation type |
| Get/SetOptionsPrefix() | set the prefix for all options |
| SetFromOptions() | customize object from command line |
| SetUp() | perform other initialization |
| View() | view the object |
| Destroy() | cleanup object allocation |

Also, all objects support the -help option.

## PETSc Options

## Ways to set options

Command line
Filename in the third argument of PetscInitialize()
~ / . petscrc
\$PWD/. petscrc
\$PWD/petscrc
PetscOptionsInsertFile()
PetscOptionsInsertString()
PETSC_OPTIONS environment variable
command line option -options_file [file]

## PETSc Options

## Example of Command Line Control

```
$> ./ex5 -da_grid_x 10 -da_grid_y 10 -par 6.7
    -snes_monitor -{ksp,snes}_converged_reason
    -snes_view
$> ./ex5 -da_grid_x 10 -da_grid_y 10 -par 6.7
    -snes_monitor -{ksp,snes}_converged_reason
    -snes_view -mat_view_draw -draw_pause 0.5
$> ./ex5 -da_grid_x 10 -da_grid_y 10 -par 6.7
    -snes_monitor -{ksp,snes}_converged_reason
    -snes_view -mat_view_draw -draw_pause 0.5
    -pc_type lu -pc_factor_mat_ordering_type natural
```

Use -help to find other ordering types

## PETSc

## Application Integration

## Application Integration

## Be willing to experiment with algorithms

No optimality without interplay between physics and algorithmics

Adopt flexible, extensible programming
Algorithms and data structures not hardwired

Be willing to play with the real code
Toy models have limited usefulness
But make test cases that run quickly

If possible, profile before integration
Automatic in PETSc

## Incorporating PETSc into Existing Codes

PETSc does not seize main (), does not control output
Propogates errors from underlying packages, flexible
Nothing special about MPI_Comm_world
Can wrap existing data structures/algorithms
MatShell, PCShell, full implementations
VecCreateMPIWithArray()
MatCreateSeqAIJWithArrays()
Use an existing semi-implicit solver as a preconditioner Usually worthwhile to use native PETSc data structures unless you have a good reason not to
Uniform interfaces across languages
C, C++, Fortran 77/90, Python, MATLAB
Do not have to use high level interfaces (e.g. SNES, TS, DM) but PETSc can offer more if you do, like MFFD and SNES Test

## Integration Stages

## Version Control

It is impossible to overemphasize
Initialization
Linking to PETSc
Profiling
Profile before changing
Also incorporate command line processing
Linear Algebra
First PETSc data structures
Solvers
Very easy after linear algebra is integrated

## Initialization

## Call PetscInitialize()

Setup static data and services
Setup MPI if it is not already
Can set PETSC_COMM_WORLD to use your communicator (can always use subcommunicators for each object)

Call PetscFinalize()
Calculates logging summary
Can check for leaks/unused options
Shutdown and release resources
Can only initialize PETSc once

## PETSc Application Integration

## Sparse Matrices

The important data type when solving PDEs
Two main phases:
Filling with entries (assembly)
Application of its action (e.g. SpMV)


## Matrix Memory Preallocation

PETSc sparse matrices are dynamic data structures can add additional nonzeros freely

Dynamically adding many nonzeros requires additional memory allocations requires copies can kill performance

Memory preallocation provides the freedom of dynamic data structures good performance

Easiest solution is to replicate the assembly code Remove computation, but preserve the indexing code Store set of columns for each row

Call preallocation routines for all datatypes
MatSeqAIJSetPreallocation ()
MatMPIBAIJSetPreallocation()
Only the relevant data will be used

## PETSc Application Integration

## Sequential Sparse Matrices

MatSeqAIJSetPreallocation (Mat $A$, int $n z$, int $n n z[])$
$n z$ : expected number of nonzeros in any row nnz(i): expected number of nonzeros in row i


## PETSc Application Integration

## Parallel Sparse Matrix

Each process locally owns a submatrix of contiguous global rows Each submatrix consists of diagonal and off-diagonal parts


MatGetOwnershipRange (Mat A, int *start,int *end)
start: first locally owned row of global matrix end-1: last locally owned row of global matrix

## PETSc Application Integration

## Parallel Sparse Matrix

```
MatMPIAIJSetPreallocation(Mat A, int dnz, int dnnz[],
    int onz, int onnz[]
```

dnz: expected number of nonzeros in any row in the diagonal block dnnz(i): expected number of nonzeros in row in the diagonal block onz: expected number of nonzeros in any row in the offdiagonal portion onnz(i): expected number of nonzeros in row $i$ in the offdiagonal portion

## PETSc Application Integration

## Verifying Preallocation

Use runtime options

```
-mat_new_nonzero_location_err
-mat_new_nonzero_allocation_err
```

Use runtime option
-info
Output:

```
[proc #] Matrix size: %d X %d; storage space: %d unneeded, %d used
[proc #] Number of mallocs during MatSetValues( ) is %d
```

[merlin] mpirun ex2 -log_info
[0]MatAssemblyEnd_SeqAIJ:Matrix size: $56 \times 56$; storage space:
[0] 310 unneeded, 250 used
[0] MatAssemblyEnd_SeqAIJ : Number of mallocs during Matsetvalues () is 0
[0] MatAssemblyEnd_SeqAIJ : Most nonzeros in any row is 5
[0]Mat_AIJ_CheckInode: Found 56 nodes out of 56 rows. Not using Inode routines
[0]Mat_AIJ_CheckInode: Found 56 nodes out of 56 rows. Not using Inode routines
Norm of error 0.000156044 iterations 6
[0]PetscFinalize:PETSc successfully ended!

## Block and Symmetric Formats

## BAIJ

Like AlJ, but uses static block size
Preallocation is like AIJ, but just one index per block

## SBAIJ

Only stores upper triangular part
Preallocation needs number of nonzeros in upper triangular parts of on- and off-diagonal blocks

## MatSetValuesBlocked()

Better performance with blocked formats
Also works with scalar formats, if MatSetBlockSize () was called
Variants MatSetValuesBlockedLocal(),
MatSetValuesBlockedStencil()
Change matrix format at runtime, don't need to touch assembly code

## Matrices

Definition (Matrix)
A matrix is a linear transformation between finite dimensional vector spaces.

Definition (Forming a matrix)
Forming or assembling a matrix means defining it's action in terms of entries (usually stored in a sparse format).

## Matrices

## Important Matrices

1. Sparse (e.g. discretization of a PDE operator)
2. Inverse of anything interesting $B=A^{-1}$
3. Jacobian of a nonlinear function $J y=\lim _{\epsilon \rightarrow 0} \frac{F(x+\epsilon y)-F(x)}{\epsilon}$
4. Fourier transform $\mathcal{F}, \mathcal{F}^{-1}$
5. Other fast transforms, e.g. Fast Multipole Method
6. Low rank correction $B=A+u v^{T}$
7. Schur complement $S=D-C A^{-1} B$
8. Tensor product $A=\sum_{e} A_{x}^{e} \otimes A_{y}^{e} \otimes A_{z}^{e}$
9. Linearization of a few steps of an explicit integrator

## Matrices

## Important Matrices

1. Sparse (e.g. discretization of a PDE operator)
2. Inverse of anything interesting $B=A^{-1}$
3. Jacobian of a nonlinear function $J y=\lim _{\epsilon \rightarrow 0} \frac{F(x+\epsilon y)-F(x)}{\epsilon}$
4. Fourier transform $\mathcal{F}, \mathcal{F}^{-1}$
5. Other fast transforms, e.g. Fast Multipole Method
6. Low rank correction $B=A+u v^{T}$
7. Schur complement $S=D-C A^{-1} B$
8. Tensor product $A=\sum_{e} A_{x}^{e} \otimes A_{y}^{e} \otimes A_{z}^{e}$
9. Linearization of a few steps of an explicit integrator

These matrices are dense. Never form them.

## Matrices

## Important Matrices

1. Sparse (e.g. discretization of a PDE operator)
2. Inverse of anything interesting $B=A^{-1}$
3. Jacobian of a nonlinear function $J y=\lim _{\epsilon \rightarrow 0} \frac{F(x+\epsilon y)-F(x)}{\epsilon}$
4. Fourier transform $\mathcal{F}, \mathcal{F}^{-1}$
5. Other fast transforms, e.g. Fast Multipole Method
6. Low rank correction $B=A+u v^{T}$
7. Schur complement $S=D-C A^{-1} B$
8. Tensor product $A=\sum_{e} A_{x}^{e} \otimes A_{y}^{e} \otimes A_{z}^{e}$
9. Linearization of a few steps of an explicit integrator

These are not very sparse. Don't form them.

## Matrices

## Important Matrices

1. Sparse (e.g. discretization of a PDE operator)
2. Inverse of anything interesting $B=A^{-1}$
3. Jacobian of a nonlinear function $J y=\lim _{\epsilon \rightarrow 0} \frac{F(x+\epsilon y)-F(x)}{\epsilon}$
4. Fourier transform $\mathcal{F}, \mathcal{F}^{-1}$
5. Other fast transforms, e.g. Fast Multipole Method
6. Low rank correction $B=A+u v^{T}$
7. Schur complement $S=D-C A^{-1} B$
8. Tensor product $A=\sum_{e} A_{x}^{e} \otimes A_{y}^{e} \otimes A_{z}^{e}$
9. Linearization of a few steps of an explicit integrator

None of these matrices "have entries"

## Matrices

What can we do with a matrix that doesn't have entries?

Krylov solvers for $A x=b$
Krylov subspace: $\left\{b, A b, A^{2} b, A^{3} b, \ldots\right\}$
Convergence rate depends on the spectral properties of the matrix For any popular Krylov method $\mathcal{K}$, there is a matrix of size $m$, such that $\mathcal{K}$ outperforms all other methods by a factor at least $\mathcal{O}(\sqrt{m})$ [Nachtigal et. al., 1992]

Typically...
The action $y \leftarrow A x$ can be computed in $\mathcal{O}(m)$
Aside from matrix multiply, the $n^{\text {th }}$ iteration requires at most $\mathcal{O}(m n)$

## GMRES

## Brute force minimization of residual in $\left\{b, A b, A^{2} b, \ldots\right\}$

1. Use Arnoldi to orthogonalize the $n$th subspace, producing

$$
A Q_{n}=Q_{n+1} H_{n}
$$

2. Minimize residual in this space by solving the overdetermined system

$$
H_{n} y_{n}=e_{1}^{(n+1)}
$$

using $Q R$-decomposition, updated cheaply at each iteration.

Properties
Converges in $n$ steps for all right hand sides if there exists a polynomial of degree $n$ such that $\left\|p_{n}(A)\right\|<t o l$ and $p_{n}(0)=1$.
Residual is monotonically decreasing, robust in practice Restarted variants are used to bound memory requirements

## PETSc Solvers

## Linear Solvers - Krylov Methods

Using PETSc linear algebra, just add:

```
KSPSetOperators(KSP ksp, Mat A, Mat M, MatStructure flag)
KSPSolve(KSP ksp, Vec b, Vec x)
```

Can access subobjects

```
KSPGetPC(KSP ksp, PC *pc)
```

Preconditioners must obey PETSc interface
Basically just the KSP interface
Can change solver dynamically from the command line, -ksp_type

Standard form of a nonlinear system

$$
F(u)=0
$$

Iteration


Solve: $\quad J(u) w=-F(u)$
Update: $\quad u^{+} \leftarrow u+w$

Quadratically convergent near a root: $\left|u^{n+1}-u^{*}\right| \in \mathcal{O}\left(\left|u^{n}-u^{*}\right|^{2}\right)$
Picard is the same operation with a different $J(u)$

## PETSc Solvers

## Nonlinear Solvers - Newton and Picard Methods

Using PETSc linear algebra, just add:

```
SNESSetFunction(SNES snes, Vec r, residualFunc, void *ctx)
SNESSetJacobian(SNES snes, Mat A, Mat M, jacFunc, void *ctx)
SNESSolve(SNES snes, Vec b, Vec x)
```

Can access subobjects

```
SNESGetKSP(SNES snes, KSP *ksp)
```

Can customize subobjects from the cmd line
Set the subdomain preconditioner to ILU with -sub_pc_type ilu

## PETSc

## Profiling

## PETSc Profiling

## Profiling

Use -log_summary for a performance profile
Event timing
Event flops
Memory usage
MPI messages
Call PetscLogStagePush() and PetscLogStagePop() User can add new stages
Call PetscLogEventBegin() and PetscLogEventEnd() User can add new events
Call PetscLogFlops () to include your flops

## PETSc Profiling

## Reading -log_summary

|  | Max | Max/Min | Avg | Total |
| :--- | :---: | :---: | :---: | :---: |
| Time (sec): | $1.548 \mathrm{e}+02$ | 1.00122 | $1.547 \mathrm{e}+02$ |  |
| Objects: | $1.028 \mathrm{e}+03$ | 1.00000 | $1.028 \mathrm{e}+03$ |  |
| Flops: | $1.519 \mathrm{e}+10$ | 1.01953 | $1.505 \mathrm{e}+10$ | $1.204 \mathrm{e}+11$ |
| Flops/sec: | $9.814 \mathrm{e}+07$ | 1.01829 | $9.727 \mathrm{e}+07$ | $7.782 \mathrm{e}+08$ |
| MPI Messages: | $8.854 \mathrm{e}+03$ | 1.00556 | $8.819 \mathrm{e}+03$ | $7.055 \mathrm{e}+04$ |
| MPI Message Lengths: | $1.936 \mathrm{e}+08$ | 1.00950 | $2.185 \mathrm{e}+04$ | $1.541 \mathrm{e}+09$ |
| MPI Reductions: | $2.799 \mathrm{e}+03$ | 1.00000 |  |  |

Also a summary per stage
Memory usage per stage (based on when it was allocated) Time, messages, reductions, balance, flops per event per stage Always send -log_summary when asking performance questions on mailing list

## PETSc Profiling



## PETSc Profiling

## Communication Costs

Reductions: usually part of Krylov method, latency limited

```
    VecDot
```

    VecMDot
    VecNorm
    MatAssemblyBegin
    Change algorithm (e.g. IBCGS)
    Point-to-point (nearest neighbor), latency or bandwidth
VecScatter
MatMult
PCApply
MatAssembly
SNESFunctionEval
SNESJacobianEval
Compute subdomain boundary fluxes redundantly
Ghost exchange for all fields at once
Better partition

## PETSc

PETSc and GPUs

## GPUs: Disillusion

Computing Architecture Schematic


## GPUs: Disillusion

## Computing Architecture Schematic



Good for large FLOP-intensive tasks, high memory bandwidth PCI-Express can be a bottleneck
$\gg 10$-fold speedups (usually) not backed by hardware

## GPU Programming Approaches

## CUDA

Almost no additional code required
Vendor-lock
Relies on nvcc being available

OpenCL
Additional boilerplate code required (low-level API)
Broad hardware support (separate SDKs)
No more development effort from NVIDIA

Directives
Annotate existing code with OpenMP-style Pragmas
OpenACC and others

## PETSc GPU Support

## NVIDIA Cusp/Thrust/CUSPARSE

Compile PETSc with CUDA support
Use command line options to enable types, e.g.

```
-vec_type cusp -mat_type aijcusp
```


## ViennaCL (OpenCL)

Compile PETSc with OpenCL support
Use command line options to enable types, e.g.

```
-vec_type viennacl -mat_type aijviennacl
```

Used for subsequent benchmarks

No change in application code required!

## Benchmarks



## Benchmarks



## Conclusions

## PETSc can help You

solve algebraic and DAE problems in your application area rapidly develop efficient parallel code, can start from examples develop new solution methods and data structures debug and analyze performance advice on software design, solution algorithms, and performance

```
petsc-{users,dev,maint}@mcs.anl.gov
```


## You can help PETSc

report bugs and inconsistencies, or if you think there is a better way tell us if the documentation is inconsistent or unclear consider developing new algebraic methods as plugins, contribute if your idea works

